ratio of satellite volume to daughter drop sizes sets the position of the high peak observed at small drop sizes. Best results were obtained if this ratio was set so as to give the mode of the peak at approximately 30 μ for all systems studied.

ACKNOWLEDGMENT

The authors express appreciation to the National Science Foundation for a grant to support research described in this paper.

NOTATION

C, C_1, C_2 = constants
D = drop diameter, L
D_{qp} = weighted average drop size as described in Equation (6), D_{32} diameter obtained with $q = 3$, $p = 2$, L
D_{max} = maximum stable drop diameter, L
D_{95} = diameter below which 95% of the volume is found, L
$D_{1.0}$ = diameter at which probability of breakup becomes 100%, L
F = force
f = arbitrary function in Equation (6)
L = length
M = mass
N_{Re} = Reynolds number based on pipe diameter and continuous phase properties
$(N_{We})_{crit}$ = critical Weber number, $\frac{\rho \overline{v^2} D_{max}}{\sigma}$
N_{Vi} = viscosity group, $\mu_d/(\rho_d \sigma D)^{1/2}$
R = tube radius, L
R_v = range of satellite to daughter drop volume ratios, L
T = time
U = average velocity, L/T
$\overline{v^2}$ = time average value of square of turbulent velocity fluctuation, L/T
y = radial distance from the pipe wall, L

Greek Letters

ε = energy dissipation per unit mass per unit time, FL/MT
μ = coefficient of viscosity, M/LT
σ = interfacial tension, L/T
ρ = density, M/L^3
Ψ = arbitrary function in Equation (1)

Subscripts

c = continuous phase
d = dispersed phase

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Feedback Control of an Enriching Column

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The transient behavior of a 24-in. diameter, 10-tray enriching column was investigated experimentally while the column was operated under closed-loop, feedback control. This behavior was compared with that of a perturbated linear model with the column open-loop behavior being represented with simple transfer functions in the frequency domain. Comparisons showed that control parameters determined with the predictive linear model were conservative and permitted stable and smooth responses of the experimental column.

At present, control systems for chemical processes can be well-designed if the process dynamics are known in either the time or frequency domain. For distillation columns, the equations describing the transient behavior are nonlinear. A numerical, nonlinear model in the time domain representing the open-loop behavior has been developed by Huckaba (*5*) and applied to open-loop studies. This and similar numerical methods involve complex numerical techniques which do not provide a simple method of studying the closed-loop behavior for control studies, and are not capable of generalizing the behavior of various columns through steady state parameters. These problems indicate more promise for modeling of the system in the frequency domain.

Simple correlations by Gilliland and Mohr (*6*) and by Pigford (*10*) have attempted to generalize the frequency response using the steady state parameters of the system. At present, however, neither method has been directly applied to control. Rosenbrock (*11*) has developed a complex numerical method employing a discrete variable time to formu-

Dr. Jack A. Gerster is deceased.

late the frequency response. Lamb, Pigford, and Rippin (7) have shown that a distillation column can be simulated by a set of linearized perturbated equations developed from the basic nonlinear equations of the column. These equations solved in the frequency domain produce the frequency responses of the column.

The linearized, perturbated equations have been shown to predict successfully the open-loop transient response of a 2-ft. diameter experimental column having five or ten trays (1). The success of the transient work led to a study to develop generalized correlations for the frequency response of columns (12). A control study employing the frequency response method was made by Haden (4) using a feedback system to control a stripping column, and the results obtained were encouraging. The present study extends the procedure to an enriching column and compares theoretical control with experimental control implemented on a large-scale column.

The experimental equipment was the same as that used and described by Luyben, Verneuil, and Gerster (8) for studying open-loop transient response. For this study, the feed is added directly into the reboiler which acts as a total vaporizer, and the bottoms product is removed from the base of the column. The response of the column was studied for upsets in the reflux rate and the feed composition while maintaining a constant overhead purity.

CONTROL SYSTEM DEVELOPMENT

The objective of the control system was to satisfy a control criterion placed on the measured variable, distillate composition, when the system is subjected to changes in feed composition and reflux rate. To effect the desirable control, either a direct measurement of the measured variable was required or a variable related directly to the distillate composition must be measured. Since the system operated at constant pressure, the liquid temperature and liquid composition were directly related, and the temperature was used as a means of measuring composition.

To maintain the measured variable constant, the vapor rate was selected as the manipulated variable. Operating at constant pressure, the feed composition, reflux ratio, and overhead product purity completely specify the state of the system. For a reflux rate change with constant overhead product purity and constant feed composition, the vapor rate was varied to maintain the reflux ratio invariant. A new value of the reflux ratio permitting the column to maintain a constant measured variable (distillate composition) was attained by correctly varying the vapor rate for a feed composition change.

For a reflux rate change, the theoretical temperature and composition profiles for the initial and final steady states are identical. Therefore temperature deviations measured on any tray could be used to represent the temperature deviations of the measured variable. For a feed composition disturbance, the initial and final steady state profiles were different, but temperature deviations on any tray if referenced to the final steady state could still represent deviations in the measured variable.

For best control, it was desirable that the tray which was most sensitive to input disturbances be used to represent the variations of the measured variable. For a given internal reflux ratio, changes in feed composition produced a series of new temperature profiles. The magnitude of the deviation per tray between a final controlled state and the initial state is plotted for various feed composition changes in Figure 1. The magnitude of these deviations can be regarded as the maximum available error per tray for control purposes. The lower trays in the column are observed to be more sensitive to a given disturbance than the trays near the top of the column. Tray 2 was selected as the control

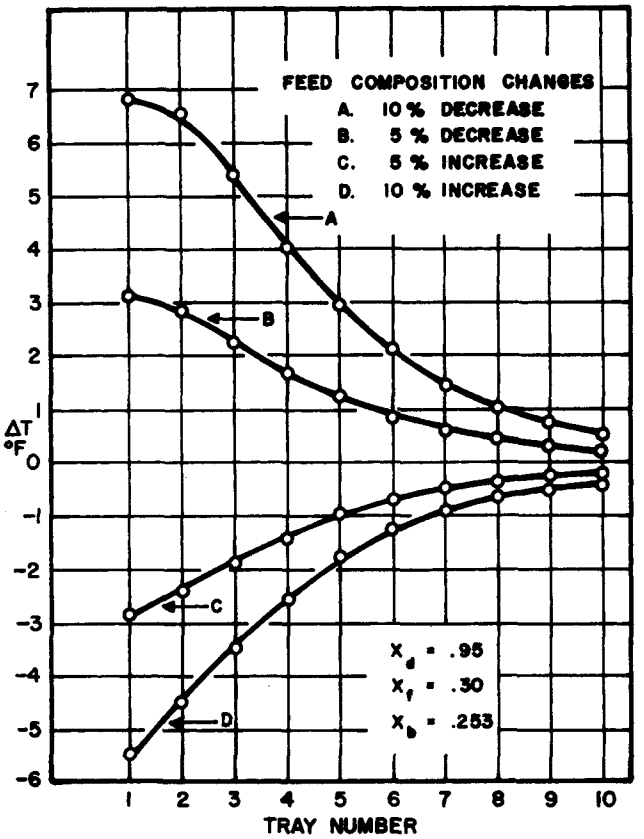

Fig. 1. Uncontrolled temperature deviations versus tray number for four changes in feed composition.

tray for this system because it had a high sensitivity relative to other trays in the column; tray 1 was eliminated as it was subject to splashing effects of the seal pan and possible random disturbances from the reboiler and base of the column.

The basic control system developed using the temperature on tray 2 as the control variable, the vapor rate as the manipulated variable, and the overhead product purity as the measured variable was a cascaded feedback control system with simple feedforward components used when feed composition changes occurred. The temperature of tray 2 was monitored continuously and fed to a primary controller. The primary controller responded to temperature deviations by varying the magnitude of the steam rate. The steam rate was maintained at the desired magnitude by a secondary controller. Both controllers were used with proportional and reset modes. Derivative action was disregarded based on work done by Haden (4).

The steam rate was related directly to the vapor rate in the column completing the control cycle. A diagram of the control system is shown in Figure 2. For reflux rate changes, the temperature on the control tray was maintained at a constant value. In the case of a feed composition change, the control tray temperature of the final controlled state was different from the temperature of the initial state, and a feedforward control component was required. For the experimental work of this study, the magnitude of the change in steady state temperatures was computed prior to a run and the change in set-point temperature for the control tray was made manually after 30 sec. to 1 min. had elapsed from the time of the original disturbance. Dynamic response of the feedforward component was eliminated based on the work of Luyben (9).

The control tray temperature was measured by a Honeywell Type A resistance thermometer used with a Moore Products Model 7711 Electro-pneumatic transducer. The

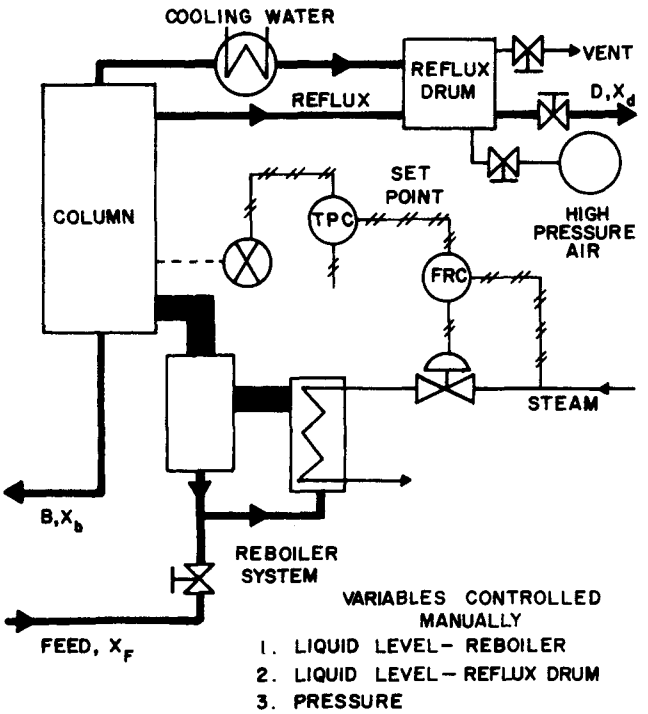

Fig. 2. Schematic drawing of enriching column and feedback control components.

temperature was recorded with a Moore Products Model 5310 recorder calibrated for a span of 20°F. and from which temperatures could be read to ±0.2°F. The steam flow was measured by an orifice meter used with a Foxboro differential pressure cell to transmit a pneumatic signal to a Moore Products Model 5310 recorder. The master control station was a Moore control station, Model 524MP5T2, and it was used in conjunction with two controllers, Moore Products Model 50M proportional plus reset units as the temperature and steam flow controllers. By calibration of the controller settings, it was found that the actual controller gain was approximately 50% less than the controller dial indicated; however, reset times varied ±15% from the indication on the controller dial.

EXPERIMENTAL WORK

The experimental work was performed on a 24-in. I.D. column containing 10 bubble-cap trays spaced at 18-in. intervals. Each tray contained 3-in. round bubble caps on 4- to 5-in. triangular spacing and had a 2-in. outlet weir and segmental downpipes. A mixture of acetone and benzene was fed to the base of a vertical, thermosyphon reboiler containing 163 tubes, each ¾ in. O.D. by 12 ft. long. The reboiler totally vaporized the liquid and it passed into the base of the tower. The liquid from the bottom tray flowed from the downpipe into a seal pan and then left the base of the column as the bottoms product. The liquid in the base of the column was in direct contact with the entering vapor.

The column was operated at pressure of 9 to 12 lb./sq. in. gauge; a nitrogen cylinder was connected to the reflux drum to counteract the small vapor leakage from the system. A large surge volume was added over the reflux drum to help minimize pressure variations created by varying the vapor rate. A vent to the atmosphere and a connection to a high-pressure nitrogen tank were both connected to the system through manually operated valves to provide manual pressure control should large pressure deviations be observed.

The range of feed compositions was from 0.288 to 0.388 mole fraction acetone and the overhead product purities ranged from 0.749 to 0.923 mole fraction acetone. Vapor rates ranged from 1.34 to 1.89 moles/min.

Reflux rate changes were made by manual manipulation of a valve near the reflux rate rotameter. Feed composition changes were made by switching from a feed tank containing liquid of one composition to a second feed tank with liquid of different composition by use of two solenoid valves. During each run, composition samples were taken of the overhead vapor, reflux, feed, bottoms product, tray 2, and the odd-numbered trays. These samples were taken at 15-sec. intervals for an initial 2½ min. period after which the interval increased to 30 sec., 1 min., or 2 min. Temperature on tray 2 and the steam rate were recorded at the control station as continuous functions.

Six runs were made in which the feed composition to the column was changed in a step function and five runs were made for which the reflux rate to the column was similarly varied. The controller settings were based on two types of analyses. In one case, four runs were based on settings found by Haden (*4*) to be experimentally optimum settings for this same column operated as a stripper. The primary settings were a gain of unity and a reset time of 2 min.; the secondary settings were a gain of 5 and a reset time of 0.3 min. In the second case four runs were made using settings based on the tuning procedure of Ziegler and Nichols (*14*) applied to the experimental column. The primary settings were a gain of from 3.85 to 5 and a reset of 0.33 min.; the secondary settings determined by this procedure were a gain of 4.17 and a reset time of either 0.1 min. or the maximum reset action attainable. The primary settings were also determined by the procedures of Cohen and Coon (*2*) and were found to be practically the same as those predicted by Zeigler and Nichols. The lower gain of 3.85 was used for the primary setting of the latter set of four runs.

Controller settings for the remaining three runs were varied slightly to observe their possible effects on the response of the system.

SIMULATION

A theoretical study of the system was made using the frequency response calculated from linearized perturbated equations to characterize the dynamic behavior of the system. The frequency response was approximated by combinations of simple transfer functions and these were used with models of the auxiliary systems and of the control hardware to form a complete simulation of the column.

The rectifying column was described by a set of linear, perturbated differential equations with coefficients dependent on the steady state. The equations for the column trays are given by Lamb, Pigford, and Rippin (*7*) and special equations were written for the seal pan, condensing system, and the top tray on which both mass and energy transfer occurred. Converting to the frequency domain, with ω the frequency in radians per minute, the equations can be solved for the relationship between any two variables as a function of the frequency, the data being represented as Bode plots.

The transfer functions of interest are represented by a process matrix **P** which relates a three-component response vector **y** to a three-component input vector **x**. The process matrix is complex and a function of the frequency; each element is an independent transfer function relating two variables of the system. The input vector **x** consists of the feed composition, reflux rate, and the internal vapor rate. The distillate rate, distillate composition, and control tray composition are the components of the response vector **y**.

$$\mathbf{y} = \begin{bmatrix} \zeta_d \\ X_d \\ X_N \end{bmatrix}; \qquad \mathbf{X} = \begin{bmatrix} X_F \\ \zeta_R \\ \nu \end{bmatrix}$$

$$\mathbf{y} = \mathbf{P}\,\mathbf{X}$$

The computation procedure for **P** requires the definition of of a matrix **G**, the auxiliary matrix, which relates the response vector **Z** consisting of the feed composition, the manipulated variable, and the control variable to the **v** vector, which consists of the measured variable, the distillate flow rate, and the reflux flow rate.

$$\mathbf{Z} = \begin{bmatrix} X_F \\ \nu \\ X_N \end{bmatrix}; \qquad \mathbf{V} = \begin{bmatrix} X_s \\ d \\ \zeta_d \end{bmatrix}$$

$$\mathbf{Z} = \mathbf{G}\,\mathbf{v}$$

To compute the j^{th} column elements of the **G** matrix, the j^{th} component of the vector **v** is set equal to unity and the remaining elements to zero. The column equations are solved in the frequency domain for a range of frequencies to determine the j^{th} column elements. The calculation is repeated until all elements of the **G** matrix have been computed; algebraic manipulations yield the **P** matrix.

The process matrix was computed for all experimental runs using average steady state values for the various coefficients. Differences in the process matrix using the initial, final, or average steady state values for the coefficients in the linear equations were small and the average values could be used with confidence. The steady state values were evaluated from equations representing McCabe-Thiele procedures in which tray efficiencies were predicted by the American Institute of Chemical Engineers procedure (3).

The transfer functions p_{ij} were plotted as Bode plots, all being referenced to zero decibel as the frequency approached zero. The transfer functions were fitted by combinations of first-order leads and first-order lags. The criteria used for fitting the transfer functions were determined by investigating the response of the system for various combinations of simple transfer functions. It was found that a fit of ±10 db. for the magnitude ratio and ±40° phase lag yield good responses differing insignificantly from curves fitted to ±2 db. Curves fitted with the latter criterion were based on the limiting slope of the magnitude plot as it approached −60 db. and the asymptotic value of the phase lag. A generally accepted limiting frequency is that which corresponds to a magnitude ratio of −60 db., and the time constant corresponding to this limiting frequency is defined as the minimum time constant. It was determined that if the time constant of any transfer component or dead time was less than the minimum value, the function could be disregarded without altering the responses of the system.

Appearing in some transfer functions were "mounds" at low frequencies attaining maximum values of +2 to +10 db. These were fitted to within ±1 db. by combining a second-order lead with two first-order lags, all with a common time constant. The damping coefficient of the second-order lead provided a means of varying the maximum of the mound. No difference in the response was observed between the complex transfer function and a simple function disregarding the "mound."

The transfer function relating the control variable to the manipulated variable is shown in Figure 3. The mound between the frequencies 0.001 and 0.01 was disregarded and the remaining curve is well-approximated by a first-order lag fitting the data to ±2 db. and a phase lag correspondence of ±30°. The average time constant for all runs is 14 min.

The transfer function relating the control variable to the feed composition was represented by a first-order lag with an average time constant of 5.3 min. for all runs. The fit is ±4 db. on the Bode plot and ±20° for the phase lag plot. The data and transfer function approximation are shown in Figure 4.

The transfer function relating the control variable to the reflux rate is shown in Figure 5. The transfer function consists of a primary first-order lag with an average time constant of 14 min. and a minor first-order lag to the third power with an average time constant of 15 sec. This time constant is close to the limiting time constant, but the function is required to match the limiting slope of the data as the frequency increases.

A block diagram of the entire system is shown in Figure 6. The measuring devices (orifice meter and resistance bulb) and the steam valve are assumed to have very fast dynamic behaviors with respect to the rest of the system and are assumed to be simple proportional elements in the system. The steam side of the reboiler was assumed to consist of a transportation lag of 12 sec. plus a first-order lag with a time constant of 12 sec. Data taken by Luyben (8) were previously interpreted as a pure transportation lag; however, this study and theory suggest a first-order lag with a small time constant combined with a small transportation lag. Part of this transportation lag could be the re-

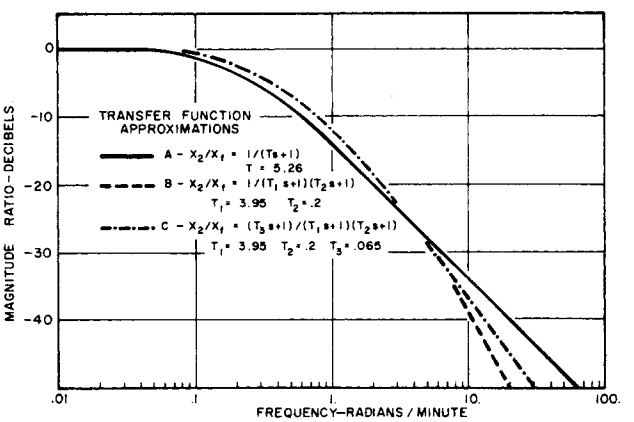

Fig. 4. Magnitude ratio versus frequency for function relating control tray composition to the feed composition (approximations).

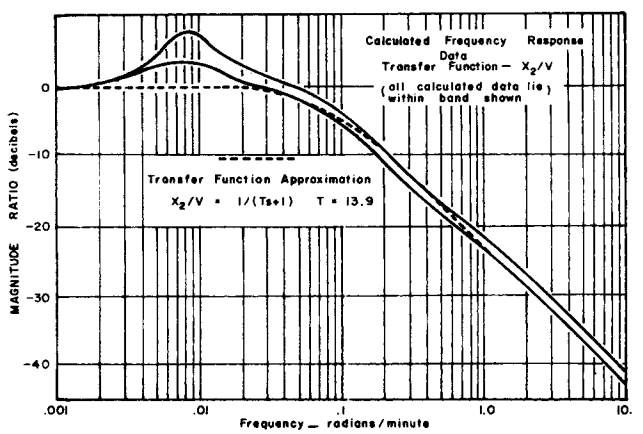

Fig. 3. Magnitude ratio versus frequency for transfer function relating control tray composition to the manipulated variable.

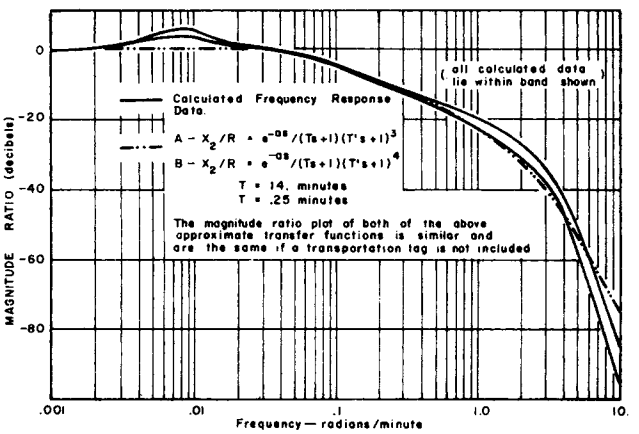

Fig. 5. Magnitude ratio versus frequency for transfer function relating control tray composition to the reflux ratio.

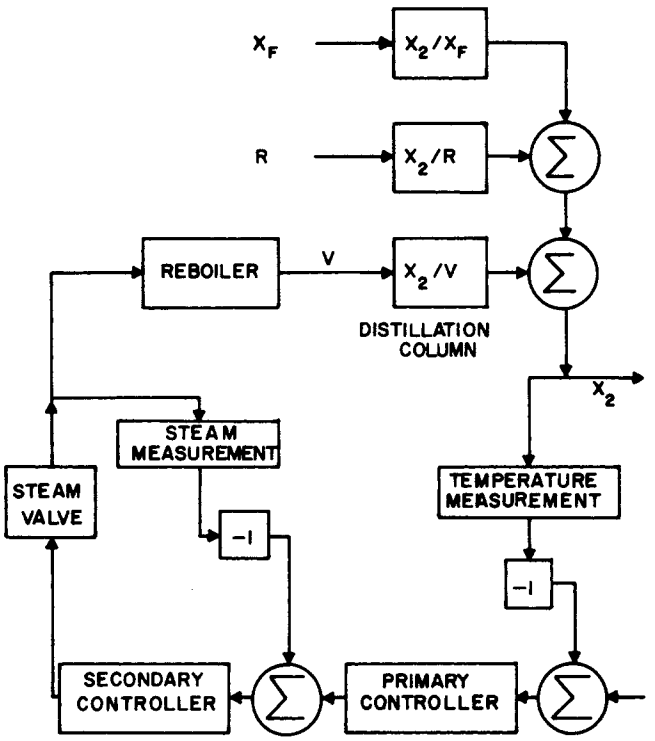

Fig. 6. Block diagram of column and control system.

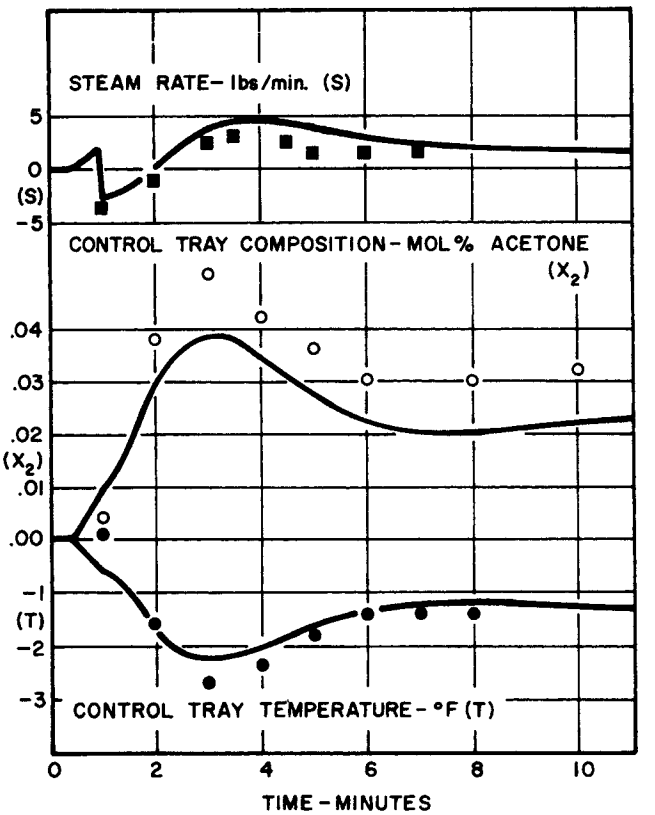

Fig. 7. Theoretical and experimental data, run 101.

sult of a dead zone in the valve because of the lack of a valve positioner. The two time constants were determined from stability considerations of the simulated model; the combination function does match the original experimental data. The equations from which the frequency response was calculated did not contain equations for the process (solvent) side of the reboiler. Therefore for feed composition changes, a pure delay was included to account for the transportation lag from the solenoid switches to the reboiler and a first-order lag for the reboiler side dynamics. Since this reboiler time constant was assumed small, both effects were approximated by delaying the introduction of the feed change by an average of 15 sec. after time zero for each run.

RESULTS

Figures 7 and 8 show the comparisons between the predicted responses and experimental data for two experimental runs in which the feed composition was varied. For run 101, the prediction of the temperature variation follows the experimental trend with time very well, although magnitude differences attain values of 15%. For run 102, the data tended to cycle after 6 min., but early behavior was predicted quite well. In both cases the manipulated variable, steam flow rate, followed the experimental data nicely. The controller gain employed was 1.0 and the reset time was 2 min. Variation of the gain did not alter the agreement between experimental and predicted responses markedly; however, a decrease in the reset time by a factor of 3.0 to 0.33 min. resulted in a predicted response that was highly unstable. The experimental response, Figure 9, did oscillate with a large amplitude, but exhibited a definite damping trend. By increasing the reset time to 1.0 min., which was the value obtained by employing the Ziegler-Nichols tuning procedure to the simulated model, the predicted response remained unstable, but agreement with the experimental results in the period of oscillation was obtained. An increase in the reset time experimentally in run 111, Figure 10, resulted in good agreement between

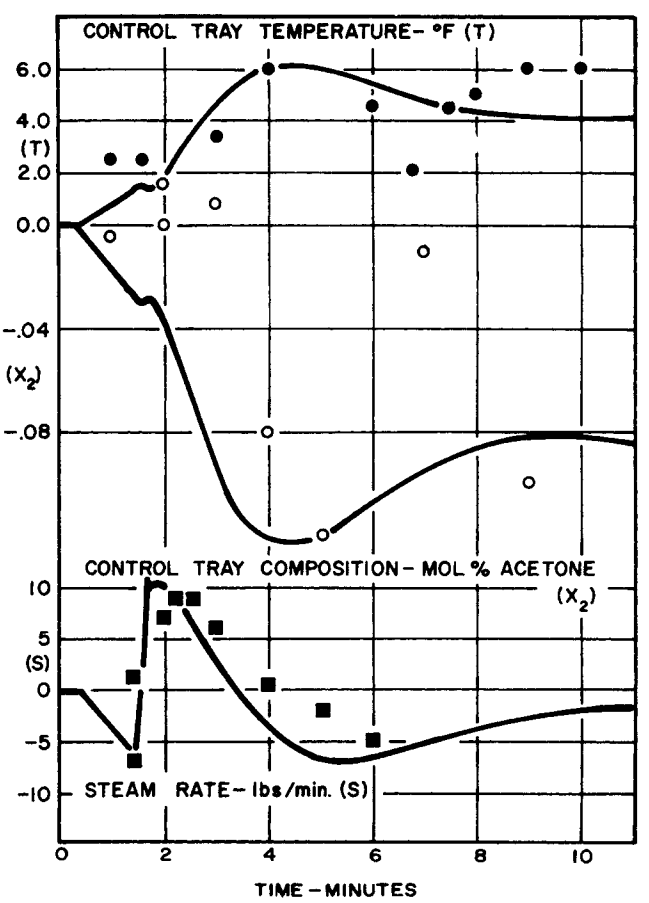

Fig. 8. Theoretical and experimental data, run 102.

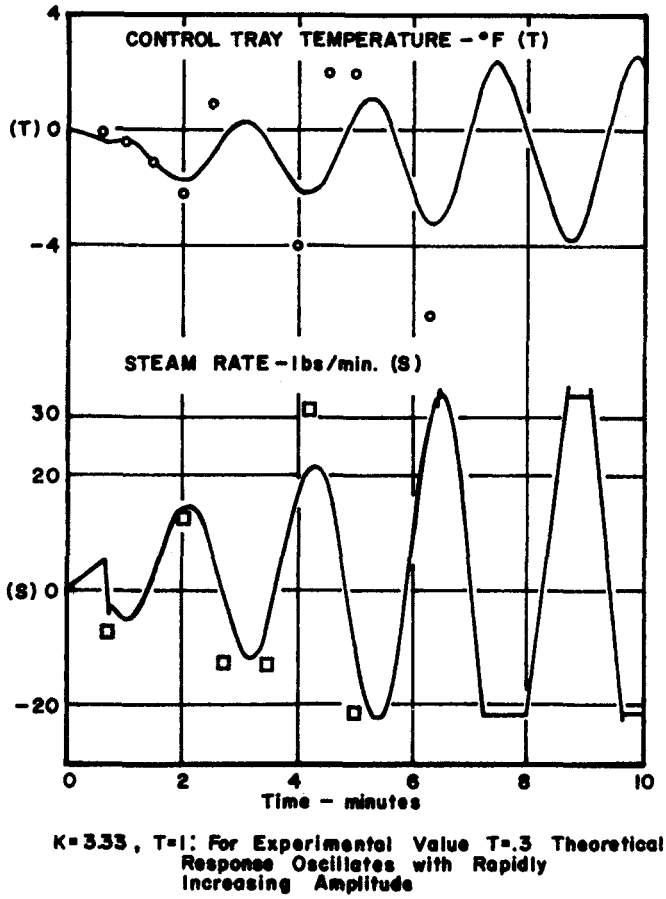

Fig. 9. Theoretical and experimental data, run 108. $K = 3.33$, $T = 1$, for experimental value $T = 0.3$. Theoretical response oscillates with rapidly increasing amplitude.

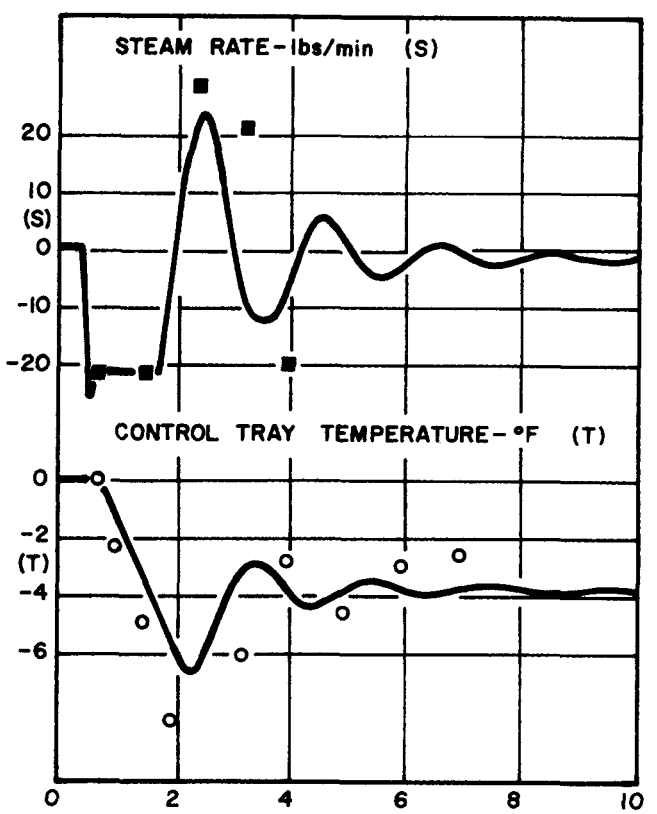

Fig. 10. Theoretical and experimental data, run 111.

the predicted and experimental responses, both being quite stable. For runs 108 and 111, the error between predicted and experimental temperature responses sometimes approached 30%; however, pressure fluctuations of ±2 lb./sq in. abs. were observed during these runs. These pressure fluctuations were created by manual pressure control and caused jumps in the experimental temperature profiles.

Reflux rate changes were made in runs 103 and 104, Figures 11 and 12. In both cases agreement was good for the steam rate which exhibited final steady state errors of 10%. Temperature profile errors were within the limits of the ex-

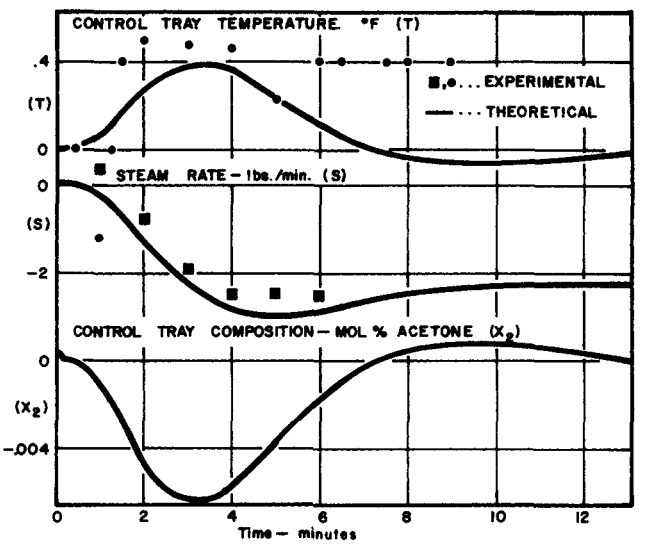

Fig. 11. Theoretical and experimental data, run 103.

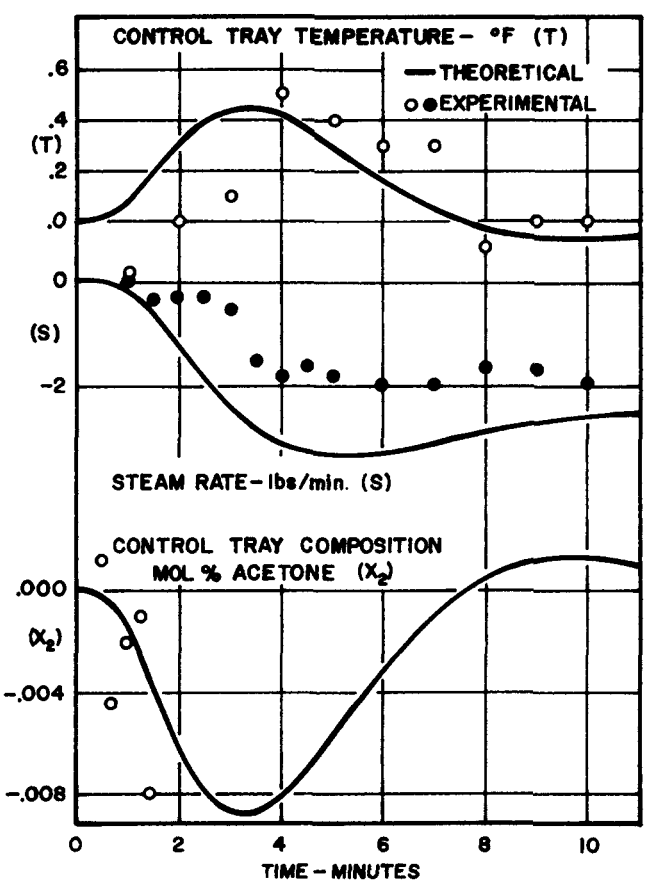

Fig. 12. Theoretical and experimental data, run 104.

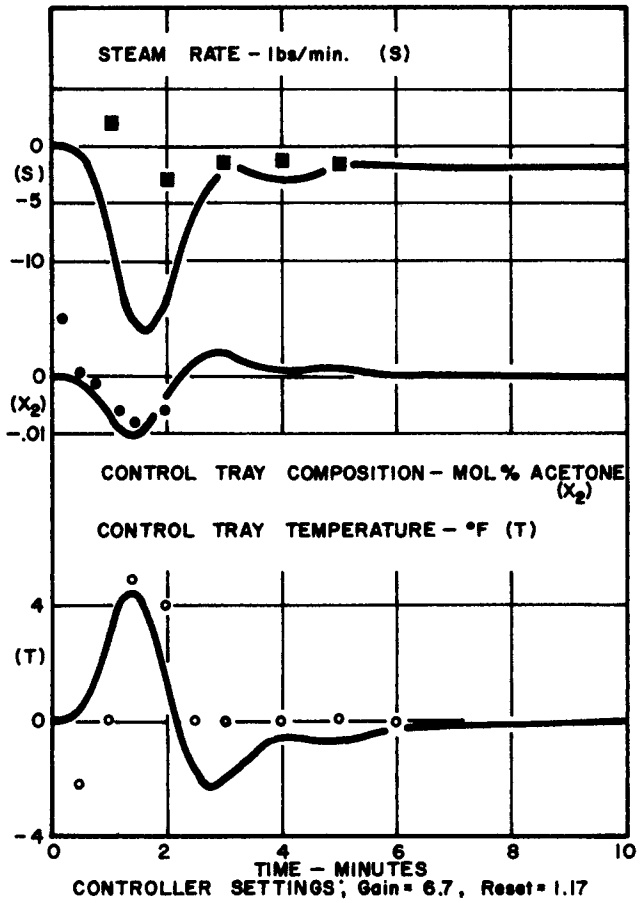

Fig. 13. Theoretical predictions, run 109. Controller settings: gain = 6.7; reset = 1.17.

perimental error of the measuring device, control system, and temperature recorder. Both responses were very stable using a primary controller gain of 1.0 and a reset time of 2 min. Again as in the case of feed variation runs, reduction of the reset time to 0.33 min., as predicted by employing the Ziegler-Nichols tuning procedure to the experimental column, resulted in a simulated response which was entirely unstable. However, if the tuning procedure was applied to the simulated model and those settings employed (a gain of 7.0 and a reset time of 1.2 min.), the predicted response closely matched the experimental behavior, see run 109, Figure 13.

CONCLUSIONS

The time period for return of the perturbated system to a steady operating condition was the critical factor in the comparison of the predicted and experimental responses. If the time behavior of the responses was in agreement, the comparison between predicted and experimental behavior was considered good. The perturbation error magnitudes of approximately 20% may be considered acceptable since pressure variations of ±2 lb./sq. in. abs. were quite prevalent experimentally; and both the pressure and liquid levels were controlled by manual adjustment which resulted in small intermittent pressure surges.

Observations of all experimental and predicted data showed that the gains predicted by employing the Ziegler-Nichols tuning procedure to the simulated model produced gains corresponding with those experimentally. However, reset times determined on the model were considerably higher than those determined experimentally. This was reflected in the more oscillatory and unstable behavior of the simulated column in comparison with the relatively stable experimental column. The greater instability of the model is to be expected since the variable interaction effects and inherent damping characteristics of a distillation column are lost in the linearization of the nonlinear equations.

From these observations, it may be concluded that controller settings determined by applying the Ziegler-Nichols tuning procedure to the simulated column would be conservative. These conservative settings would produce smooth and sometimes overdamped responses; however, the times required for stable responses to reach a new steady state would be well predicted. This evidence indicates that a simple linearized model, which is easily implemented on an analog computer, can be employed for control studies and the prediction of transient column behavior. This results in considerable savings in both time and expense in comparison to nonlinear numerical procedures.

ACKNOWLEDGMENT

The authors express their thanks to the National Science Foundation for its financial support of this study.

NOTATION

$\mathbf{G}$ = auxiliary matrix
$\mathbf{P}$ = process matrix
$\mathbf{v}$ = input vector
$\mathbf{x}$ = input vector
x_d = distillate composition
x_F = feed composition
x_N = liquid composition on N^{th} tray
$\mathbf{y}$ = response vector
$\mathbf{z}$ = response vector

Greek Letters

ζ_d = dimensionless distillate rate
ζ_R = dimensionless reflux rate
ν = dimensionless vapor rate

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